Optimal strategies for retirees in Australia with realistic risk transfer

Adam Butt*^

Gaurav Khemka*^

Geoffrey J. Warren*

* Research School of Finance, Actuarial Studies and Statistics, Australian National University

^ Co-lead Investigator, Data61, Commonwealth Scientific and Industrial Research Organisation

28 January 2021

Abstract

We investigate optimal investment and drawdown decisions for Australian retirees, allowing for simple risk transferring options. The preferred asset mix varies significantly across different preferences, balances and home ownership. Loss aversion utility leads to hedging strategies to secure the target consumption through use of life and deferred life annuities and asset allocation and drawdown decisions. Under risk aversion utility, annuities replace defensive portfolio investment in the account-based pension, smoothing consumption and limiting its decline after account exhaustion. Our results have implications for the design of comprehensive income products for retirement in the Australian market.

Keywords

Retirement, Optimisation, Dynamic Programming

Acknowledgements

The authors would like to acknowledge the valuable feedback provided by Anthony Asher, David Bell, John de Ravin, Jeremy Duffield, Jim Hennington, Bonsoo Koo, Estelle Liu, Hugh Morrow, Deborah Ralston, Nicolette Rubinsztein, Brnic van Wyk and Shang Wu. Further, the authors would like to acknowledge the feedback received from presenting this work at the 2020 Actuaries Institute All-Actuaries Virtual Summit, the 28th Colloquium on Pensions and Retirement Research, and at various university seminar series.

I. Introduction

Retirees face complex financial decisions that many are ill-equipped to make (Agnew et al., 2013). The Australian legislative environment enforces pre-retirement saving on most income earners through the Superannuation Guarantee (SG). However, few restrictions are placed on how these savings are utilised in the decumulation phase (Henry, 2009), meaning that individuals have significant responsibility for managing their own financial risks in retirement. The vast majority self-manage these risks through investment in an account-based pension (ABP), also known as an allocated pension (Australian Prudential Regulation Authority, 2018), where consumption is financed through flexible drawdown of the retirement savings balance.

As an outcome of the 2014 Financial System Inquiry, the Australian government is in the process of requiring superannuation funds to introduce Comprehensive Income Products for Retirement (CIPRs) by 2022. The purpose of CIPRs is to provide a soft default for the post-retirement (i.e. decumulation) phase. Whilst the design requirements of CIPRs are currently vague, they should provide the opportunity to transfer more risk than using only an ABP.

This paper identifies how optimal investment and drawdown strategies for an Australian retiree under a lifecycle model[1] vary with balance, homeownership and preferences. The investment strategy involves allocating between an ABP and purchase of a life annuity[2] (LA) and/or a deferred life annuity[3] (DLA) at retirement, and then dynamically setting the growth and defensive weights within the ABP over the retirement phase. Investment in both an ABP and annuity can meet the CIPR requirement of balancing "income, risk management and flexibility" (Australian Government The Treasury, 2014). We also construct optimal drawdown strategies, subject to the minimum drawdown rules under Australian regulations. We assume access to the means-tested Age Pension, thus allowing for its interaction with investment and drawdown strategies[4].

[1] A common approach to modelling optimal financial decisions is through lifecycle models, which originated in a seminal paper by Modigliani and Brumberg (1954) and were extended by Merton (1969) and Samuelson (1969); see Kotlikoff (2008) for a summary of the early literature. The model assumes that agents maximise expected utility over their entire lifetime; and can be used to optimise key decisions such as consumption versus savings, education and workforce participation. In this paper, we consider only the post-retirement phase of life and the decisions relevant to this phase.

[2] A life annuity provides a certain income until death. Since Yaari (1965) demonstrated that it is optimal to fully annuitise in the absence of a bequest motive, there has been significant research into reasons for low annuisation rates, including Lockwood (2012) and Yogo (2016) for the U.S., and Ganegoda and Bateman (2008) and O'Meara et al. (2015) for Australia.

[3] A deferred life annuity (DLA, also known as a "longevity annuity" or "deferred income annuity") provides a certain income until death from a specified age sometime after purchase. There is a limited literature on DLAs, most which considers purchase of DLAs during the working phase to provide income payments after retirement (e.g. Horneff et al. (2010), Konicz and Mulvey (2013), and Dillschneider et al. (2019)). More recent work incorporating DLAs into lifecycle models (e.g. Iskhakov et al. (2015), Huang et al. (2017), Horneff et al. (2020), and Habib et al. (2020)) indicates it is optimal to allocate a small proportion of assets at retirement to this product.

[4] Pashchenko (2013) develops a lifecycle model to show that means-tested benefits significantly reduce annuitisation behaviour. Bütler et al. (2017) provide evidence of such behaviour for low wealth retirees in Switzerland.

This paper contributes to a small but growing literature on optimal financial decisions by retirees in an Australian context. Hulley et al. (2013) consider the optimal portfolio allocation and drawdown decisions of retirees who have access to the Age Pension and an ABP. They find that high-wealth households are incentivised to invest heavily in equities due to the hedge provided by the means-tested Age Pension while tending to draw down at a slower rate than low-wealth households, as they observe in empirical data. Ding (2014) models optimal consumption and housing decisions calibrated against Australian data; while Andréasson et al. (2017) extend Ding (2014) by including a portfolio allocation decision[5]. Andréasson and Shevchenko (2017) show that optimal drawdowns during retirement are not linear in balance as per Samuelson (1969), but impacted by the means-testing of the Age Pension. Means-testing also results in optimal risky asset weights displaying a complex non-linear relationship with balance, rather than being constant as per Merton (1969). Similar results appear in Butt et al. (2018). Iskhakov et al. (2015) consider optimal decisions of an Australian retiree with access to a LA or DLA (but not both). With no Age Pension, they show that the proportion of wealth allocated to LAs is invariant to wealth levels. Availability of the Age Pension crowds out LA purchases at lower balances because it acts as a LA. Similar patterns occur for DLAs, although they are found less attractive at higher balances and slightly more attractive at lower balances.

An important difference between this study and the literature cited above relates to the utility functions used to represent preferences. The literature uses either constant relative risk aversion (CRRA) and/or Hyperbolic Absolute Risk Aversion (HARA) utility[6]. HARA utility applies a consumption floor to the CRRA utility structure and leads to a trivial annuitisation decision that guarantees meeting the consumption floor. In addition to CRRA utility, we instead apply the value function of cumulative prospect theory (Tversky and Kahneman, 1992), referred to here as 'loss aversion' utility[7], assuming an explicit consumption target above the floor provided by the Age Pension. The target aligns with the concept of adequacy of post-retirement income, and allows consumption below the target should circumstances dictate. To the best of our knowledge, we are the first to apply loss aversion utility to identifying optimal investment and drawdown strategies in an Australian

[5] Updated results for changes to Age Pension means testing are presented in Andréasson and Shevchenko (2017). Additional extensions to the model, incorporating (separately) LA purchase and a reverse mortgage option, are presented in Andreasson and Shevchenko (2019).

[6] HARA utility is used in an Australian context by Thorp et al. (2007), Iskhakov et al. (2015), and Andréasson et al. (2017).

[7] Rabin and Thaler (2001) show that standard concave utility structures such as CRRA are inadequate for explaining economic phenomena, and that loss aversion provides a stronger rationale for these observations; whilst Abdellaoui (2000) demonstrates that individuals make decisions following the general shape of the loss aversion utility form. Loss aversion utility has been used in a range of discrete and continuous-time models to solve portfolio and/or consumption decisions, in for example, Berkelaar et al. (2004), Jin and Yu Zhou (2008), Bernard and Ghossoub (2010), Blake et al. (2013), Chen et al. (2017) Dong and Zheng (2019), and van Bilsen and Laeven (2020).

context. We also allow for a bequest motive[8], which is mostly incidental in our baseline results but becomes influential when a strong bequest motive is assumed under sensitivity testing.

We identify optimal strategies across a range of individual cameos that capture differences in balance, homeownership, and preference structures/appetites. We find that strong loss aversion leads to hedging strategies to secure the consumption target, manifesting as a meaningful initial allocation to annuities followed by subsequent asset allocation and drawdown decisions designed to deliver the target as far as feasible. Demand for annuities also emerges under CRRA utility, and replaces defensive portfolio investment in the ABP, in order to both smooth consumption and limit its decline after exhaustion of the ABP, with degree of demand being influenced by the level of risk aversion. On the other hand, minimal demand for annuities emerges under loss aversion preferences for low balances and under weak loss aversion, where the optimal strategy is to invest in growth assets in order to achieve the target and seek gains, respectively. Finally, due to the Age Pension asset means-test, high initial drawdowns from the ABP are observed for those with larger initial assets and either weak loss aversion or CRRA preferences, in order to receive a larger Age Pension in future.

The paper is structured as follows. Section II outlines the methodology. Section III presents the results and discussion. Section IV concludes. An Appendix provides additional detail on the methodology.

II. Methodology

We estimate optimal decisions for Australian retirees using stochastic dynamic programming within a lifecycle model. The model is similar to that used in Butt et al. (2019), with adjustments to allow for the availability of annuities, up-to-date Age Pension rules, and the ability to examine individuals that differ by account balance, homeownership status and preferences. We model a single male who retires at age 67 and earns no further labour income. The individual funds their consumption during retirement through drawdowns from an account-based pension (ABP), income from the purchase of a life annuity (LA) and/or deferred life annuity (DLA), and receipt of the Age Pension and related supplements. The retiree may choose to invest their ABP across growth and defensive assets. Purchase of the LA and DLA can be made at age 67 only[9], with income from the DLA commencing at age 85. The remainder of this section outlines the model and solution approach.

[8] Cho and Sane (2013) show that means-testing arrangements cause Australian retirees not to utilise housing wealth for retirement consumption, which is consistent with the treatment of housing in our model.

[9] Retirees in Andreasson and Shevchenko (2019) may only annuitise once, but this can be at any age. They find that annuitisation at retirement age gives similar results to annuitisation at later ages. Thus this simplifying assumption should not have a material impact on results, whilst greatly simplifying the optimisation model.

a) Optimisation model

The model is expressed in discrete time $t \in [0,1,2,\ldots,42]$ measured in years, with age equivalently being 67+t for all t. At $t > 0$, decisions are made to optimise lifetime utility V_t, with the Bellman equation as follows[10]:

$$V_t = \max_{D_t,\pi_t}\left[U_{i,t} + \mathrm{E}\left[p_t V_{t+1} + q_t B_{i,t+1}\right] | A_t, H_t, L, DL\right] \quad (1)$$

where D_t is the drawdown from the account-based pension; π_t is the proportion of the ABP invested in growth assets; $U_{i,t}$ is utility generated from consumption for utility structure i; p_t is the probability that an individual aged 67+t will be alive at age 67+t+1; $q_t = 1 - p_t$ is the probability that an individual aged 67+t will die before age 67+t+1; $B_{i,t}$ is utility generated from bequests for utility structure i; A_t is the balance of the ABP before drawdown; H_t is the home value; L is the purchase price of the LA at $t = 0$; and DL is the purchase price of the DLA at $t = 0$.

At $t = 0$, the retiree also decides how much L and DL to purchase, with the Bellman equations as follows:

$$V_0 = \max_{D_0,\pi_0,L,DL}\left[U_{i,0} + \mathrm{E}\left[p_0 V_1 + q_0 B_{i,1}\right] | A_0, H_0\right] \quad (2)$$

The following transfer functions apply:

$$A_{t+1} = (A_t - D_t - I_{t=0}[L + DL])\left(1 + \pi_t r_{t,g} + (1 - \pi_t) r_{t,d}\right) \quad (3)$$

$$C_t = D_t + \frac{L}{a_L} + I_{t\geq 18}\frac{DL}{a_{DL}} + P_t \quad (4)$$

$$H_{t+1} = H_t\left(1 + r_{t,h}\right) \quad (5)$$

where I_j is an indicator function equal to 1 if condition j is met or 0 otherwise; $r_{t,g}$ is the percentage return on the growth portfolio; $r_{t,d}$ is the percentage return on the defensive portfolio; C_t is consumption; a_L is a pricing factor used to determine the income from the LA; a_{DL} is a pricing factor used to determine the income from the DLA; P_t is the Age Pension received; and $r_{t,h}$ is the percentage increase in home value.

Definitions, calculations and constraints relevant to these functions, parameters and variables are outlined in the sub-sections below. Further modelling details appear in the Appendix.

b) Utility functions

We consider two utility structures $U_{i,t}$, both of which include a bequest term $B_{i,t}$ as per Lockwood (2018)[11]. The first utility structure is loss aversion utility ($i = LAU$). It uses the value function of cumulative prospect theory

[10] We assume that there is no subjective discount factor for intertemporal utility preferences. Huang et al. (2012) note the difficulty in determining a subjective discount factor where decisions are highly impacted by lifetime uncertainty.

[11] Allowing for a bequest motive supports a coherent comparison between homeowners and renters, given that we do not allow homeowners to utilise the value of their property in providing for retirement consumption through, for example, a reverse mortgage.

(Tversky and Kahneman, 1992) to estimate expected utility, eschewing the editing and probability distortion components of prospect theory. We apply budget-based consumption targets (Chybalski and Marcinkiewicz, 2016) as described in Section II(c). The structure under loss aversion utility is as follows:

$$U_{LAU,t} = I_{(C_t \geq C_t^*)}(C_t - C_t^*)^{\alpha} - I_{(C_t < C_t^*)}\lambda(C_t^* - C_t)^{\beta} \quad (6)$$

$$B_{LAU,t} = \left(\frac{\phi}{1-\phi}\right)^{1-\alpha}(A_t + H_t)^{\alpha} \quad (7)$$

where C_t^* is the consumption target; α is the curvature parameter in respect of gains; β is the curvature parameter in respect of losses; λ is loss aversion; and ϕ is the strength of the bequest motive[12]. This formulation treats bequests by evaluating them as a form of above-target consumption, and is consistent with the treatment of bequests under constant relative risk aversion (CRRA) utility as outlined below.

The second utility structure of CRRA utility (i = *CRRA*) assumes no specific target, but rather a concern with maximising utility arising directly from the level of consumption and bequest outcomes. Its functional form is as follows:

$$U_{CRRA,t} = \frac{C_t^{1-\rho}}{1-\rho} \quad (8)$$

$$B_{CRRA,t} = \left(\frac{\phi}{1-\phi}\right)^{\rho}\left(\frac{A_t+H_t}{1-\rho}\right)^{1-\rho} \quad (9)$$

where ρ is the coefficient of relative risk aversion.

c) Cameos and utility calibration

The cameos considered are outlined in Table 1, with selected parameter values appearing in the table notes. The choice of cameos is explained in Appendix (a).

d) Other assumptions

Minimum drawdown rule restrictions on D_t and details on the Age Pension P_t, can be found in Appendix (b). Since borrowing or short selling are generally not permitted within superannuation funds, we apply additional constraints of $D_t \leq A_t$, $0 \leq \pi_t \leq 1$; $0 \leq L$; $0 \leq DL$; and $0 \leq L + DL \leq A_0$.

Mortality is based on the male rates in the Australian Life Tables 2015-17 (Australian Government Actuary, 2019) assuming 125 year improvement factors. Individuals are assumed to die with certainty by age 110. We vary this in sensitivity testing to investigate a fixed time horizon for retirement.

---

[12] In Equation (1), utility from bequest is expressed in terms of time t+1, implying that bequests are transferred at the end of the year of death.

Table 1 – Cameos considered in the optimisation model

Cameo	**Initial Balance A_0**	**House Value H_0**	**Utility $U_{i,t}$**	**Consumption Target C_t^***
1. Low wealth, renter, modest consumption target	\$200,000	Nil	Loss aversion s. BWZ w. TK	Modest – Housing Cost + Rent
2. Low wealth, homeowner, modest consumption target	\$200,000	\$341,160	Loss aversion s. BWZ w. TK	Modest
3. Average wealth, renter, comfortable consumption target	\$500,000	Nil	Loss aversion s. BWZ w. TK	Comfortable – Housing Cost + Rent
4. Average wealth, homeowner, comfortable consumption target	\$500,000	\$631,045	Loss aversion s. BWZ w. TK	Comfortable
5. High wealth, homeowner, comfortable consumption target	\$800,000	\$900,017	Loss aversion s. BWZ w. TK	Comfortable
6. Average wealth, homeowner, no consumption target	\$500,000	\$631,045	CRRA s. ρ = 5 w. ρ = 2	N/A
7. High wealth, homeowner, no consumption target	\$800,000	\$900,017	CRRA s. ρ = 5 w. ρ = 2	N/A

- "s" = strong and "w" = weak parameter settings of both loss and risk aversion
- BWZ is the parameters of Blake et al. (2013), giving $\alpha = 0.44$; $\beta = 0.88$, $\lambda = 4.50$
- TK is the parameters of Tversky and Kahneman (1992), giving $\alpha = 0.88$; $\beta = 0.88$, $\lambda = 2.25$
- $\phi = 0.5$ is used for all cameos, and is varied to impose a strong bequest motive in sensitivity testing
- Modest target is \$27,913 or \$26,450 where $t < 19$ or $t \geq 19$ respectively (in $t = 0$ dollars); Housing Cost is \$4,128
- Comfortable target is \$43,787 or \$41,613 where $t < 19$ or $t \geq 19$ respectively (in $t = 0$ dollars); Housing Cost is \$5,026
- Rent (R_t) is 5.35% of H_t for Low wealth and 3.83% of H_t for Average wealth

We model in real terms with respect to Consumer Price Index (CPI), but allow home prices, rents and the basic Age Pension (not supplements and rental assistance) to grow in line with real wages, which we set at 1.5%[13]. Returns for the growth ($r_{t,g}$) and defensive ($r_{t,d}$) portfolios consist of 135 observations that are used in forming expectations under Equations (1) and (2). The growth portfolio has a compound real return of 4.5% per annum with standard deviation 15.0% per annum, while the defensive portfolio has a compound real return of 1.0% per annum with standard deviation 4.4% per annum. These are varied in sensitivity testing. Further information and justification can be found in Appendix (c).

Annuity pricing is based on quotes received from Challenger Limited effective at 31 December 2018, of annuities equivalent to those available in this model[14]. The annuity quote was an income of \$5,596 for the LA and \$21,291 for the DLA per \$100,000 in purchase price, which is equivalent to $a_L = 17.86991$ and $a_{DL} = 4.69682$ from Equation (4). These are varied in sensitivity testing.

[13] Hence $r_{t,h}$ from Equation (5) is equal to 1.5%.

[14] The yield on long-term Australian government inflation linked securities at 31 December 2018 was 0.8% per annum, which is similar to the expected return of 1.0% per annum on the defensive portfolio and hence justifies the appropriateness of these quotes.

The consumption targets expressed in the notes to Table 1 are in $t = 0$ dollars. For future t we project forward these targets by calculating the extent to which they exceed the basic rate of the Age Pension at $t = 0$, and add this difference to the indexed level of basic Age Pension. This retains a constant gap between the consumption target and Age Pension in CPI-deflated terms over time. An important consequence of this assumption is that, despite annuities being indexed at a lower level than the consumption target, it is still possible for individuals to purchase annuities to guarantee the difference between the Age Pension and consumption target.

e) Solution approach

Optimal decisions are determined recursively from Equations (1) and (2). On first pass through, the ABP balance state variable A_t and the purchase prices of both L and DL are discretised in $10,000 increments. Since H_t is deterministic and no stochastic transfer function applies to L and DL, the stochastic element of the expectation in Equations (1) and (2) is only across A_t, and so single-variable interpolation as per Butt et al. (2019) can be undertaken. Optimisation of D_t and π_t in Equation (1) is undertaken using R (R Core Team, 2019) with the DEoptim package (Price et al., 2006). In Equation (2) (i.e. at $t = 0$), D_t and π_t are similarly optimised for all potential discretised combinations of A_0, L and DL, with the optimal L and DL being determined by a global search for the values that solve Equation (2). On second pass through, and given the optimal L and DL from the first pass through, D_t and π_t are optimised again using $1,000 increments of A_t.

Using these optimal decisions, we perform 10,000 simulations for each cameo based on random draws with replacement from the growth and defensive portfolio return sample outlined in Section II(d), and projecting from age 67 to age 110.

III. Results

All results in this section are expressed in real (i.e. CPI-deflated) terms.

(a) Summary results

Table 2 presents summary results. We report the optimal initial allocation to account-based pension (ABP), life annuity (LA) and deferred life annuity (DLA) products at age 67; the income arising from the LA and DLA (from age 85); and the median age at which the ABP is exhausted.

Low wealth renters (Cameo 1) cannot meet the modest consumption target through annuity purchase, and so choose to seek returns via the ABP in hope of reaching the target. Cameo 1w with weak loss aversion is more willing to accept consumption below the target, and their ABP lasts until a later age compared to Cameo 1s with strong loss aversion.

Table 2 – Summary results from cameos

Cameo	Parameter Setting	Initial Balance	Initial Balance Allocation: Account-Based Pension	Initial Balance Allocation: Life Annuity Purchase *L*	Initial Balance Allocation: Deferred Life Annuity Purchase *DL*	Annuity Income: Life Annuity Income L / a_L	Annuity Income: Deferred Life Annuity Income DL / a_{DL}	Median age at which $A_t = 0$
1. Low wealth, renter, modest consumption target	s. BWZ	$200,000	$200,000 (100%)	-	-	-	-	88
	w. TK	$200,000	$200,000 (100%)	-	-	-	-	91
2. Low wealth, homeowner, modest consumption target	s. BWZ	$200,000	$180,000 (90%)	-	$20,000 (10%)	-	$4,258	96
	w. TK	$200,000	$200,000 (100%)	-	-	-	-	105
3. Average wealth, renter, comfortable consumption target	s. BWZ	$500,000	$500,000 (100%)	-	-	-	-	88
	w. TK	$500,000	$500,000 (100%)	-	-	-	-	90
4. Average wealth, homeowner, comfortable consumption target	s. BWZ	$500,000	$190,000 (38%)	$260,000 (52%)	$50,000 (10%)	$14,550	$10,646	94
	w. TK	$500,000	$460,000 (92%)	-	$40,000 (8%)	-	$8,516	99
5. High wealth, homeowner, comfortable consumption target	s. BWZ	$800,000	$600,000 (75%)	$90,000 (11%)	$110,000 (14%)	$5,036	$23,420	94
	w. TK	$800,000	$800,000 (100%)	-	-	-	-	101
6. Average wealth, homeowner, no consumption target	s. $\rho = 5$	$500,000	$320,000 (64%)	$120,000 (24%)	$60,000 (12%)	$6,715	$12,775	92
	w. $\rho = 2$	$500,000	$420,000 (84%)	-	$80,000 (16%)	-	$17,031	91
7. High wealth, homeowner, no consumption target	s. $\rho = 5$	$800,000	$550,000 (69%)	$130,000 (16%)	$120,000 (15%)	$7,275	$25,549	92
	w. $\rho = 2$	$800,000	$680,000 (85%)	-	$120,000 (15%)	-	$25,549	91

See Table 1 for a full description of each cameo. Initial Balance Allocation is the optimal decision from Equation (2). Annuity Income is amount of income arising from the optimal annuity purchase decisions, with *L* generating an income immediately and *DL* generating an income from age 85. The median age at which account balance $A_t = 0$ comes from the simulations described in Section II(e). For some cameos it is optimal to drawdown to very close to $A_t = 0$ and then consume small proportions of the remaining balance for many years. In these cases, the value provided is the median age at which A_t is first less than 0.6% of A_0.

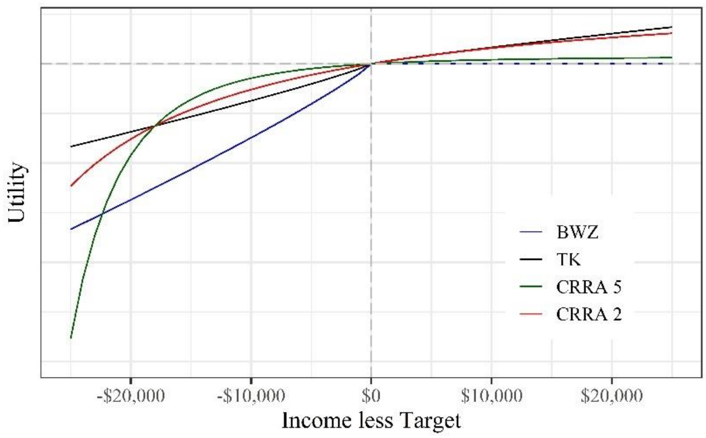
Low wealth homeowners (Cameo 2) are more capable of meeting their modest consumption target. Cameo 2s with strong loss aversion allocates 10% to a DLA to generate income of $4,258. When combined with the Age Pension, this is enough to reach age 86 with guaranteed income of $35,756 compared to a modest consumption target of $33,679. Cameo 2w with weak loss aversion obtains more utility by seeking gains above the consumption target, and hence allocates entirely to the ABP.

Cameos 3 and 4 are an average wealth renter and homeowner respectively, while Cameo 5 is a high wealth homeowner. For these cameos, the desire to purchase annuities depends on the ability to attain the comfortable consumption target and the level of loss aversion. Similar to Cameo 1, Cameo 3 cannot attain the consumption target with annuity purchase, and allocates 100% to the ABP in the hope of meeting the target.

A desire for annuity purchase emerges for Cameos 4 and 5, and is much higher for strong loss aversion (Cameos 4s, 5s) compared to weak loss aversion (Cameos 4w, 5w). For example, Cameo 4s allocates 62% to annuities that generate income of $14,550 prior age 85 and a total of $25,195 after age 85. This is sufficient when combined with the Age Pension to guarantee an income of $51,246 at age 86 compared to a comfortable consumption target at age 86 of $48,842. Cameo 5s allocates less to annuities, although with a substantially smaller purchase of the LA but a larger DLA than Cameo 4s. The DLA purchase guarantees meeting the consumption target in old age, with the larger amount placed in the ABP allowing consumption above the target in earlier years.

The annuitisation decisions across Cameos 3s, 4s and 5s can be viewed as hedging behaviour, which is most apparent where loss aversion is high and wealth and ability to meet the consumption target through annuity purchase are closely matched. Hedging behaviour is weaker where wealth is well above or well below this level. Meanwhile, those with weak loss aversion purchase little to no annuities in order to seek gains, and are willing to risk exhaustion of the ABP and hence inability to meet the target. The rationale for these results can be seen in Figure 1, which plots all utility functions used. Weak loss aversion (TK) places the lowest penalty on consumption more than $20,000 below the target, while seeing highest benefit from gains above the target. They are thus more willing to risk not meeting the consumption target to seek gains. Consumption below target is severely penalised for those with strong loss aversion (BWZ), with little utility gain for consumption above target. They thus desire strategies that minimise the possibility of losses.

Figure 1 – Utility functions, U_t, and parameter settings used



BWZ and KT lines are unadjusted with a consumption target C^* of $41,613. A linear adjustment is made to CRRA utility values so that they are on the same scale.

Cameos 6 and 7 are respectively average and high wealth homeowners with CRRA utility, which implies no consumption target but rather a desire to maximise the level of consumption accounting for risk. Under CRRA utility, similar proportions are allocated to annuities at average and high wealth, with higher allocations under strong risk aversion (Cameos 6s, 7s) than weak risk aversion (Cameos 6w, 7w). Annuities assist with smoothing consumption and secure a level of income once the ABP is exhausted. Figure 1 shows that those with strong risk aversion (ρ = 5) are more heavily impacted by very low consumption levels than those with weak risk aversion, (ρ = 2) and so purchase additional annuities to reduce uncertainty in consumption.

The drivers of our annuitisation results differ to Iskhakov et al. (2015) and Andreasson and Shevchenko (2019), and highlight the role of the utility structure and risk tolerance. Annuity demand is driven by desire to maintain the consumption floor in Iskhakov et al. (2015) and Andreasson and Shevchenko (2019). Our results show that under a consumption target rather than a floor, annuity purchase emerges as a hedging strategy to help secure the target for those with strong loss aversion. The application of this hedging strategy depends on the ability of the individual to achieve the target given their available balance[15]. Meanwhile, those with weak loss aversion have limited desire to purchase annuities, but rather choose to seek gains. Annuity demand also emerges under CRRA utility, where the prime motivation is to smooth and insure against relatively low levels of consumption – hence those with strong risk aversion purchase more annuities.

(b) Further analysis

We present a more detailed analysis of optimisation and simulation results for consumption and asset allocation for Cameos 4 and 6, through providing and discussing plots appearing as Figure 2 through to Figure 5. These cameos are both average wealth homeowners, with loss aversion and risk aversion preferences respectively. Box 1 describes the information provided in the four panels appearing in these figures.

Figure 2 presents results for Cameo 4s, representing an average wealth homeowner with strong loss aversion who aims to sustain their comfortable consumption target through allocating 62% to purchasing annuities (LA 52%, DLA 10%). Panel (a) shows consumption in line with the target at age 70 for ABP balances between \$50,000 and \$130,000, with drawdowns of only \$8,000 required to attain the target. Cameo 4s starts with \$190,000 in the ABP at age 67, which suffices to sustain the target with relative ease, and even support small additional above-target consumption, until the DLA generates income from age 85. Panel (a) also shows consumption above the target for all ABP balances at age 86, supported by DLA income. Panel (c) shows very little variation in consumption with even 10th percentile consumption being above the target at all ages.

[15] In our model, the consumption target is a desired standard of living that is assumed to be exogenously determined without reference to income that is affordable given the available balance. There is some literature on endogenous consumption targets (e.g. Polkovnichenko (2007)), although this is outside the scope of this paper and hence we leave this for future research.

Box 1 – Information provided in Figure 2 – Figure 5

Panel (a) presents optimal consumption amounts from Equations (1) and (2) split by various sources, with the ABP balance A_t presented on the x-axis. These are presented for age 70 (t = 3), which is the fourth year after the initial LA and DLA decision; and age 86 (t = 19), which is the second year that the DLA delivers an income and the first year that new parameters apply for annuities under the means-test for the Age Pension.

Panel (b) presents optimal asset allocation proportions from Equations (1) and (2) across the ABP balance, LA and DLA[16], with total assets[17] appearing on the x-axis. In the asset allocation plots, the x-axis labels start from the value of the LA and DLA (i.e. A_t equals zero at the leftmost point), which allows vertical comparisons to be made with the consumption graphs. Panel (b) plots are presented for the same ages as Panel (a).

Panel (c) presents simulated consumption amounts as described in Section II(e), split by various sources, with age appearing on the x-axis. Cross-sectional median and 10th/90th percentiles are provided[18].

Panel (d) presents simulated asset amounts as described in Section II(e), split by various sources, with age appearing on the x-axis. Cross-sectional median and 10th/90th percentiles are provided[19].

For reasons of brevity, only a selection of panels is presented for Cameos 4 and 6 only. Full results for all cameos are available in the supplementary material.

The use of defensive assets for hedging, as discussed in Section III(a), can be seen in Panel (b). At age 70, less than 6% of total assets are invested in the ABP growth portfolio where the total assets are around $400,000 (ABP balance around $125,000), but this increases to greater than 20% when total assets are around $350,000 or $450,000 (ABP balance around $75,000 or $175,000). At age 86, all ABP investment is in the growth portfolio, as the consumption target is secured with annuity income and so additional consumption can be pursued. Panel (d) shows that less than 20% of the initial $500,000 at age 67 is invested in the growth portfolio, with the allocation between the growth and defensive portfolios varying significantly across the percentiles until age 85. The 90th percentile results occur when ABP growth portfolio returns are above expectations, which permits additional investment in the growth portfolio to seek further gains due to the greater certainty in meeting the consumption target. The 10th percentile results occur when ABP growth portfolio returns are below expectations, with investment in the growth portfolio reduced to secure the consumption target.

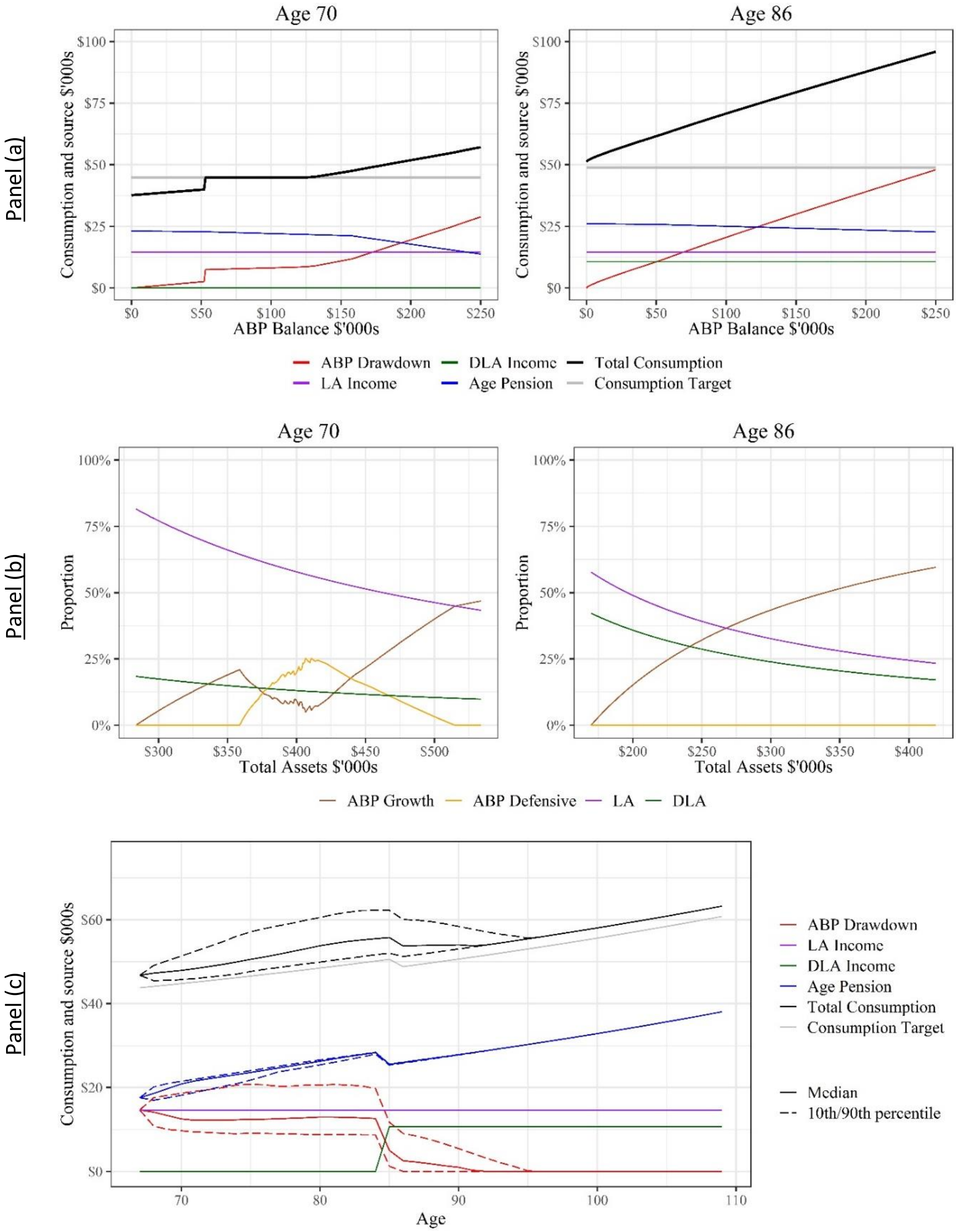
[16] The value of the LA and DLA are calculated by determining the effective discount rates based on the quoted prices, and recalculating the net present values at ages 70 and 86 using these rates. Different discount rates are applied pre and post age 85 so that the LA and DLA are valued on the same basis post age 85.

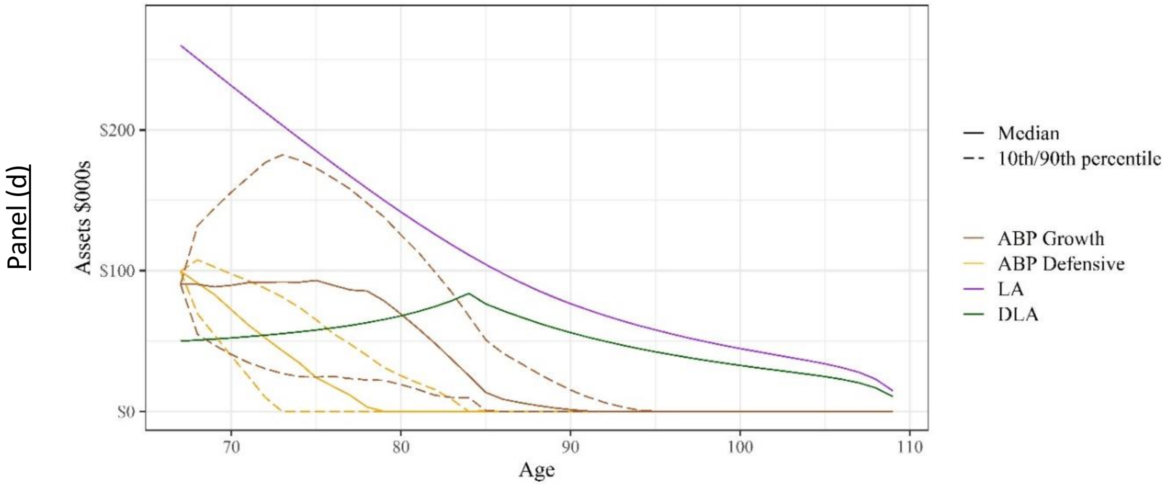
[17] We do not include the value of the Age Pension in the asset graphs. For reference, for Cameo 4s at age 67 we estimate a median simulated value for the Age Pension (discounted at the same rate as the LA) of $430,000. This compares with allocations of $190,000, $260,000 and $50,000 to ABP, LA and DLA respectively.

[18] Medians and percentiles are determined from total consumption from the cross-sectional simulation output for each age, then split between component consumption elements from that simulation at that age. For example, the 90th percentile outcomes for the Age Pension and drawdown amounts come from the same simulation, and thus can be added together with LA and DLA income to give total consumption.

[19] As above, medians and percentiles are determined from ABP from the cross-sectional simulation output for each age, then split between the component ABP elements from that simulation at that age . Since the LA and DLA amounts are determined at age 67, only a single median value is presented for these components.

Figure 2 – Cameo 4s results (Average wealth, homeowner, comfortable consumption target, BWZ)





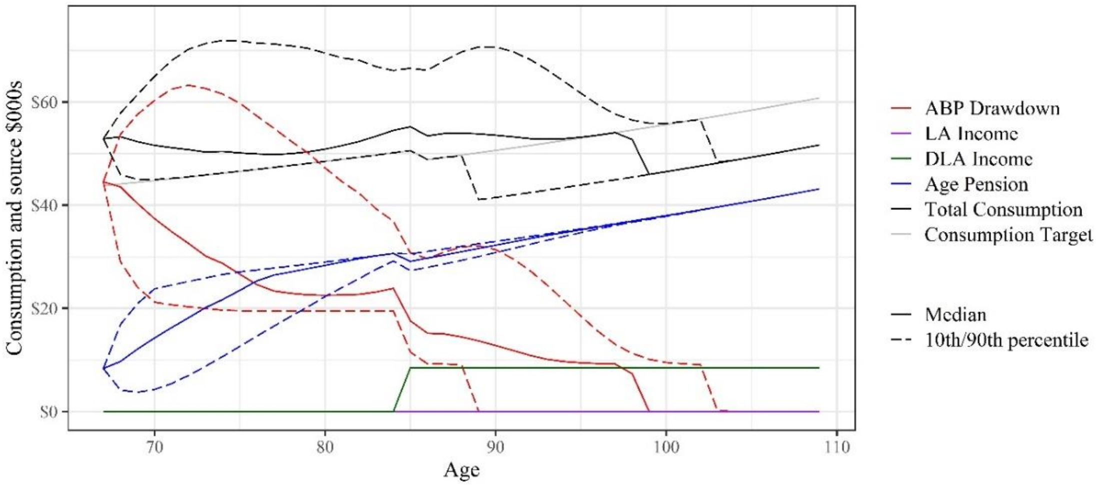
See Table 1 for a description of Cameo 4s. See Box 1 for a description of the panels.

Figure 3 presents consumption simulation outcomes for Cameo 4w, which is identical to Cameo 4s except with weak rather than strong loss aversion leading to 92% being allocated to the ABP (see Table 2). Panel (c) shows a much higher but more variable propensity to consume above the target compared to under strong loss aversion (see Figure 2, Panel (c)). This strategy comes with the risk of falling below the target, with inability to meet the consumption target upon ABP exhaustion at ages 89, 99 and 103 at the 10th, 50th and 90th percentiles respectively. The higher level of ABP means that the asset test applies under Cameo 4w, rather than the income test as was the case in Cameo 4s. This results in the Age Pension received in early years being much lower than Cameo 4s, although the initial drawdown is also much higher in Cameo 4w. Higher drawdown results in the median Age Pension being higher after age 75 in Cameo 4w, as compared to Cameo 4s. This is consistent with Asher et al. (2017), who observe higher drawdowns under the asset test consistent with the incentive to reduce the ABP balance in order to receive a higher Age Pension. It is also consistent with the modelling of Andréasson and Shevchenko (2017).

Asset allocation panels are not presented for Cameo 4w and the remaining cameos, where 100% of the ABP is allocated to the growth portfolio. Only a small allocation (8%) is made to a DLA in Cameo 4w, compared to much larger LA and DLA purchases and significant defensive portfolio investment in Cameo 4s. This highlights the enormous differences in investment strategies that can emerge for strong versus weak loss aversion for individuals with an available balance that just supports meeting their consumption target.

Figure 3 – Cameo 4w results (Average wealth, homeowner, comfortable consumption target, TK)

Panel (c)

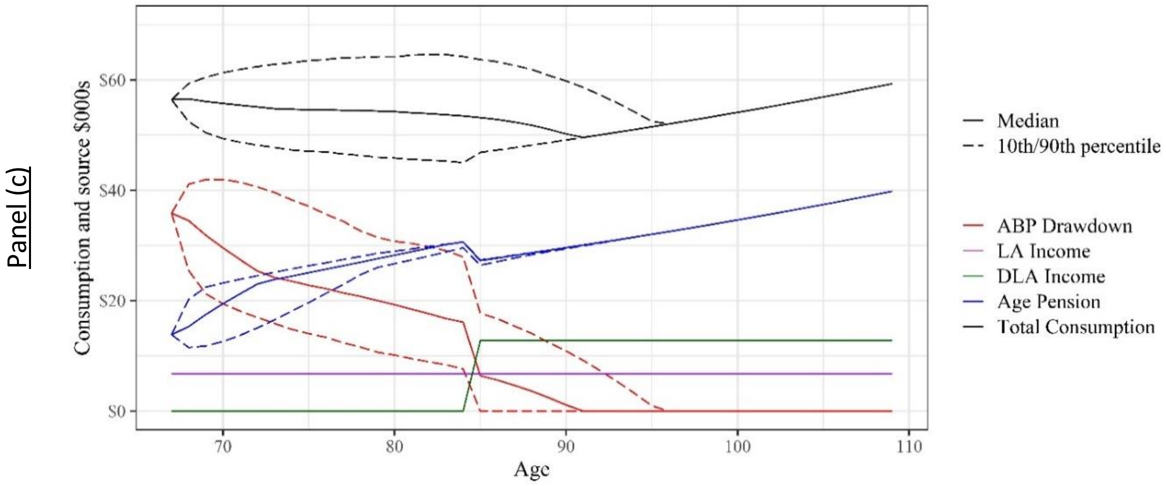


See Table 1 for a description of Cameo 4w. See Box 1 for a description of the Panel (c).

Figures 4 and 5 present strong and weak risk aversion results for Cameo 6, an average wealth homeowner with CRRA preferences. Consistent with the shape of the utility functions in Figure 1, consumption under strong risk aversion (Figure 4 Panel (c)) is smoother than under weak risk aversion (Figure 5 Panel (c)), while additional annuity purchase results in higher consumption after ABP exhaustion. Consumption starts at a higher level for weak risk aversion, reflecting the lower allocation to annuities and hence higher expected return on the portfolio. The overall slight downward slope in the median consumption curves until ABP exhaustion reflects the desire to increase Age Pension receipt through ABP drawdown (as discussed for Cameo 4w in Figure 3 Panel (c)), along with the discounting effect that mortality weighting has on overall utility calculations. Both Cameos 6s and 6w have higher median consumption in early years than Cameo 4w (Figure 3 Panel (c)), as the latter has a desire to protect the consumption target even though loss aversion is weak.

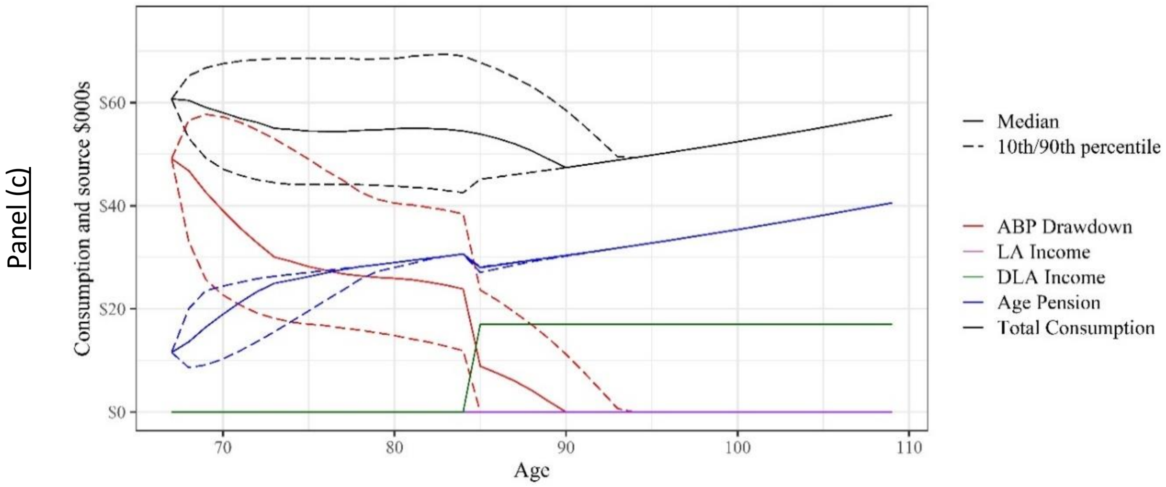
Unlike Andréasson and Shevchenko (2017) and Butt et al. (2018), the means testing of the Age Pension is found to play no role in asset allocation decisions within the ABP, with all non-annuity investment in the growth portfolio. While annuities lack flexibility after purchase, their certainty in income is more valuable than the ability to dynamically adjust asset allocation after accounting for means testing. Annuities hence crowd out defensive portfolio investment within the ABP, consistent with Iskhakov et al. (2015).

Figure 4 – Cameo 6s results (Average wealth, homeowner, CRRA ρ = 5)



See Table 1 for a description of Cameo 6s. See Box 1 for a description of the Panel (c).

Figure 5 – Cameo 6w results (Average wealth, homeowner, CRRA ρ = 2)



See Table 1 for a description of Cameo 6w. See Box 1 for a description of the Panel (c).

(c) Sensitivity testing

Box 2 outlines the sensitivity tests which are undertaken on Cameos 4 and 6.

Box 2 – Sensitivity tests undertaken on Cameos 4 and 6

(i) bequest ϕ = 0.96 investigates the impact of a significantly stronger bequest motive. The initial bequest parameter of ϕ = 0.50 (Malloy, 2015) places a relatively low weighting on bequest relative to consumption. The choice of ϕ = 0.96 is from Lockwood (2018).

(ii) lower yields are motivated by the current relatively low yields, including record low interest rates, indicating the possibility of returns being lower in future than historical averages. Defensive portfolio returns are reduced by 1% per annum, with LA and DLA prices recalculated using a 1% per annum lower discount rate. We test three adjustments to the growth portfolio; risk premium unchanged (rp0), increased by 1% per annum (rp+1) and reduced by 1% per annum (rp-1)[20].

(iii) time horizon for death at age 97 considers an individual who makes decisions for a 30-year time horizon for retirement, which has a 10% probability of occurring. Hence q_t in Equation (1) and (2) is assumed to be zero for $0 < t < 29$ and one for $t = 29$. No adjustment is made to annuity pricing.

[20] Given that defensive portfolio returns have been reduced by 1% per annum, this is equivalent to reducing growth portfolio returns by 1%, 0% and 2% for rp0, rp+1 and rp-1 respectively.

Table 3 – Summary results from sensitivity analysis of cameos

Cameo	Parameter Setting	Initial Balance	Initial Balance Allocation: Account-Based Pension	Initial Balance Allocation: Life Annuity Purchase L	Initial Balance Allocation: Deferred Life Annuity Purchase DL	Annuity Income: Life Annuity Income L / a_L	Annuity Income: Deferred Life Annuity Income DL / a_{DL}	Median age at which $A_t = 0$
4. Average wealth, homeowner, comfortable consumption target	*s. BWZ*	*\$500,000*	*\$190,000 (38%)*	*\$260,000 (52%)*	*\$50,000 (10%)*	*\$14,550*	*\$10,646*	*94*
	w. TK	*\$500,000*	*\$460,000 (92%)*	-	*\$40,000 (8%)*	-	*\$8,516*	*99*
(i) bequest ϕ = 0.96	s. BWZ	\$500,000	\$220,000 (44%)	\$220,000 (44%)	\$60,000 (12%)	\$12,311	\$12,775	95
	w. TK	\$500,000	\$500,000 (100%)	-	-	-	-	110
(ii) lower yield; risk premium unchanged (rp0)	s. BWZ	\$500,000	\$150,000 (30%)	\$310,000 (62%)	\$40,000 (8%)	\$15,565	\$6,828	101
	w. TK	\$500,000	\$450,000 (90%)	-	\$50,000 (10%)	-	\$8,535	97
(ii) lower yield; risk premium +1% p.a. (rp+1)	s. BWZ	\$500,000	\$150,000 (30%)	\$310,000 (62%)	\$40,000 (8%)	\$15,565	\$6,828	101
	w. TK	\$500,000	\$460,000 (92%)	-	\$40,000 (8%)	-	\$6,828	99
(ii) lower yield; risk premium -1% p.a. (rp-1)	s. BWZ	\$500,000	\$130,000 (26%)	\$340,000 (68%)	\$30,000 (6%)	\$17,072	\$5,121	101
	w. TK	\$500,000	\$430,000 (86%)	-	\$70,000 (14%)	-	\$11,949	95
(iii) time horizon for death at age 97	s. BWZ	\$500,000	\$170,000 (34%)	\$270,000 (54%)	\$60,000 (12%)	\$15,109	\$12,775	98
	w. TK	\$500,000	\$290,000 (58%)	-	\$210,000 (42%)	-	\$44,711	98
6. Average wealth, homeowner, no consumption target	*s. ρ = 5*	*\$500,000*	*\$320,000 (64%)*	*\$120,000 (24%)*	*\$60,000 (12%)*	*\$6,715*	*\$12,775*	*92*
	w. ρ = 2	*\$500,000*	*\$420,000 (84%)*	-	*\$80,000 (16%)*	-	*\$17,031*	*91*
(i) bequest ϕ = 0.96	s. ρ = 5	\$500,000	\$500,000 (100%)	-	-	-	-	110
	w. ρ = 2	\$500,000	\$500,000 (100%)	-	-	-	-	110
(ii) lower yield; risk premium unchanged (rp0)	s. ρ = 5	\$500,000	\$320,000 (64%)	\$130,000 (26%)	\$50,000 (10%)	\$6,527	\$8,535	92
	w. ρ = 2	\$500,000	\$450,000 (90%)	-	\$50,000 (10%)	-	\$8,535	91
(ii) lower yield; risk premium +1% p.a. (rp+1)	s. ρ = 5	\$500,000	\$360,000 (72%)	\$80,000 (16%)	\$60,000 (12%)	\$4,017	\$10,242	94
	w. ρ = 2	\$500,000	\$450,000 (90%)	-	\$50,000 (10%)	-	\$8,535	92
(ii) lower yield; risk premium -1% p.a. (rp-1)	s. ρ = 5	\$500,000	\$300,000 (60%)	\$150,000 (30%)	\$50,000 (10%)	\$7,532	\$8,535	90
	w. ρ = 2	\$500,000	\$350,000 (70%)	\$110,000 (22%)	\$40,000 (8%)	\$5,523	\$6,828	88
(iii) time horizon for death at age 97	s. ρ = 5	\$500,000	\$270,000 (54%)	\$120,000 (24%)	\$110,000 (22%)	\$6,715	\$23,420	98
	w. ρ = 2	\$500,000	\$310,000 (62%)	-	\$190,000 (38%)	-	\$40,451	98

See Table 1 for a full description of each cameo. See Table 2 for a description of results. See Box 2 for a description of sensitivities tested.

Table 3 summarises the sensitivity analysis of Cameos 4 and 6. More detailed results are presented in the supplementary material. As expected, the impact of a higher bequest motive (test (i)) shifts the initial allocation away from annuitisation. All cameos apart from 4s(i) invest 100% in the ABP, while consuming at a lower level to maintain a higher ABP balance to use as a bequest until certain death at age 110. Cameo 4s(i) involving strong loss aversion makes similar decisions to Cameo 4s. This arises because the bequest is treated as a gain, which provides little value compared to meeting consumption targets.

The impact of lower yields (test (ii)) on those with strong loss aversion in Cameo 4s(ii) is to adjust LA and DLA allocations to guarantee meeting the consumption target. More is invested in a LA and less in a DLA due to the more severe impact of lower yields on DLA pricing compared to LA pricing. Less is invested in the ABP due to the higher annuity purchase necessary to guarantee meeting the consumption target. Even less is invested in the ABP when the growth risk premium is reduced (rp-1) due to the lower returns earned on the ABP. Cameo 4w(ii) with weak loss aversion still invests the majority in the ABP growth portfolio to seek gains, but invests slightly different amounts in a DLA than Cameo 4w depending on the relative value of the LA and DLA compared to the growth portfolio. Cameos 6s(ii) and 6w(ii) adjust LA and DLA purchase relative to Cameos 6s and 6w respectively, depending on the relative value of the LA and DLA compared to the growth portfolio. Like for Cameo 4s(ii), more is invested in a LA and less in a DLA. Where the growth risk premium is unchanged (rp0) then very little change is observed in annuity purchase, whereas annuity purchase decreases when the growth risk premium increases (rp+1) and annuity purchase increases when the growth risk premium decreases (rp-1).

Planning for a retirement time horizon to age 97 (test (iii)) results in additional DLA purchase compared to the original results, due to the value it provides over the assumed 12-year payment period which now occurs with certainty. Cameos 6s(iii) and 6w(iii) with risk aversion, initially consume less than under Cameos 6s and 6w respectively, to maintain the ABP balance until age 98 due to the assumption of survival until guaranteed death at that age. Cameo 4s(iii) involving strong loss aversion makes similar decisions to Cameo 4s.

IV. Conclusion

We have investigated optimal financial decisions for an Australian retiree allowing for a range of simple risk transferring options similar to what might be used in designing Comprehensive Income Products for Retirement (CIPR). Retirees can allocate their wealth at retirement between an account-based pension (ABP), life annuity (LA) and deferred life annuity (DLA). They then dynamically set the weighting of a growth and defensive portfolio within the ABP as well as their drawdowns over the retirement phase.

Our results show how annuitisation and other asset allocation and drawdown decisions depend on preferences, and are often driven by risk hedging decisions. For those with strong loss aversion preferences, hedging through annuity purchase is most desirable where wealth is at a level that closely supports securing the consumption target, and leads to higher levels of LA as compared to DLA purchase. Those with an excess of wealth compared to that required to attain the consumption target tend to guarantee income at later ages through purchasing a DLA, while making portfolio decisions to seek further gains. This hedging continues after annuitisation until the DLA becomes available. Those not able to use annuities to secure consumption targets increase defensive exposure within the ABP if returns are good, whilst those who have used annuities to secure consumption targets increase defensive exposure within the ABP if returns are poor. On the other hand, individuals with weak loss aversion purchase virtually no annuities, instead preferring to invest in the growth portfolio to seek gains. The influence of risk hedging also emerges under CRRA preferences, where some annuities are purchased to limit the decline in consumption once the ABP is exhausted, as well as to smooth consumption. Those with strong risk aversion prefer a less risky portfolio and hence purchase a higher amount of annuities.

Looking at consumption decisions, those with strong loss aversion tend to consume at the target level across a wide range of ABP holdings, while those with weak loss aversion are more willing to consume above the target and run a risk of falling below the target if returns are poor. The impact of the asset means-test can be seen for those with larger initial assets along with either weak loss aversion or risk aversion preferences, who initially drawdown from the ABP and consume at high rates in order to receive a larger Age Pension in future. However, dynamic adjustment of asset allocation in response to the Age Pension asset means-test is not observed, as we find that annuities crowd out defensive portfolio investment for these individuals.

One particular challenge for CIPR design is trading off optimal results arising from complex modelling such as that undertaken here, and the desire for simplicity in product design. Our modelling does not allow for more complicated risk transfer products such as investment-linked and pooled annuities, nor does it allow for equity access for homeowners. To avoid complexity, one approach might be to design products around ‘rules of thumb’ that provide outcomes which are close to optimal, but are readily implemented and easily understood. We intend to investigate this issue in future, aiming to demonstrate the value from merging academic research with product design in meeting the needs of both customers and product providers.

References

Abdellaoui, M. (2000), 'Parameter-free elicitation of utility and probability weighting functions', *Management Science*, **46**, 1497-1512.

Agnew, J.R., Bateman, H. and Thorp, S. (2013), 'Financial Literacy and Retirement Planning in Australia', *Numeracy: Advancing Education in Quantitative Literacy*, **6**, 1-25.

Andreasson, J.G. and Shevchenko, P.V. (2019), 'Optimal annuitisation, housing decisions and means-tested public pension in retirement under expected utility stochastic control framework', Unpublished paper.

Andréasson, J.G. and Shevchenko, P.V. (2017), 'Assessment of Policy Changes to Means-Tested Age Pension Using the Expected Utility Model: Implication for Decisions in Retirement', *Risks*, **5**, 47.

Andréasson, J.G., Shevchenko, P.V. and Novikov, A. (2017), 'Optimal consumption, investment and housing with means-tested public pension in retirement', *Insurance: Mathematics and Economics*, **75**, 32-47.

Asher, A., Meyricke, R., Thorp, S. and Wu, S. (2017), 'Age pensioner decumulation: Responses to incentives, uncertainty and family need', *Australian Journal of Management*, **42**, 583-607.

Association of Superannuation Funds of Australia (2019), 'ASFA Retirement Standard, Detailed budget breakdowns, September quarter 2019'.

Australian Government Actuary (2019), 'Australian Life Tables 2015-17', Commonwealth of Australia.

Australian Government Productivity Commission (2020), 'PC Productivity Insights', Commonwealth of Australia.

Australian Government The Treasury (2014), 'Financial System Inquiry, Final Report, November 2014', Commonwealth of Australia.

Australian Prudential Regulation Authority (2018), 'Annual Superannuation Bulletin June 2018'.

Australian Taxation Office (2020), 'Minimum annual payments for super income streams'. https://www.ato.gov.au/rates/key-superannuation-rates-and-thresholds/?page=10. (Accessed on 8 January 2020).

Azar, S.A. (2006), 'Measuring relative risk aversion', *Applied Financial Economics Letters*, **2**, 341-345.

Berkelaar, A.B., Kouwenberg, R. and Post, T. (2004), 'Optimal portfolio choice under loss aversion', *Rev Econ Stat*, **86**, 973-987.

Bernard, C. and Ghossoub, M. (2010), 'Static portfolio choice under cumulative prospect theory', *Mathematics and Financial Economics*, **2**, 277-306.

Blake, D., Wright, D. and Zhang, Y.M. (2013), 'Target-driven investing: Optimal investment strategies in defined contribution pension plans under loss aversion', *Journal of Economic Dynamics & Control*, **37**, 195-209.

Bütler, M., Peijnenburg, K. and Staubli, S. (2017), 'How much do means-tested benefits reduce the demand for annuities?', *J Pension Econ Finan*, **16**, 419-449.

Butt, A., Khemka, G. and Strickland, L. (2018), 'How Academic Research Can Inform Default Superannuation Fund Design and Individual Financial Decsion-Making', *The Australasian Journal of Applied Finance*, **1**, 40-49.

Butt, A., Khemka, G. and Warren, G. (2019), 'What dividend imputation means for retirement savers', *Economic Record*, **95**, 181-199.

Chen, Z., Li, Z., Zeng, Y. and Sun, J. (2017), 'Asset allocation under loss aversion and minimum performance constraint in a DC pension plan with inflation risk', *Insurance: Mathematics and Economics*, **75**, 137-150.

Cho, S.-W.S. and Sane, R. (2013), 'Means-tested age pensions and homeownership: is there a link?', *Macroeconomic Dynamics*, **17**, 1281-1310.

Chybalski, F. and Marcinkiewicz, E. (2016), 'The Replacement Rate: An Imperfect Indicator of Pension Adequacy in Cross-Country Analyses', *Social Indicators Research*, **126**, 99-117.

Clare, R. (2019), 'Better Retirement Outcomes: a snapshot of account balances in Australia', Association of Superannuation Funds of Australia.

Dillschneider, Y., Maurer, R. and Schober, P. (2019), 'Dynamic Portfolio Choice with Annuities When the Interest Rate Is Stochastic', Unpublished paper.

Dimson, E., Marsh, P. and Staunton, M. (2019), 'Summary Edition Credit Suisse Global Investment Returns Yearbook 2019', Credit Suisse Research Institute.

Ding, J. (2014), 'Essays on post-retirement financial planning and pension policy modelling in Australia', PhD Thesis, Macquarie University.

Dong, Y. and Zheng, H. (2019), 'Optimal investment of DC pension plan under short-selling constraints and portfolio insurance', *Insurance: Mathematics and Economics*, **85**, 47-59.

Ganegoda, A. and Bateman, H. (2008), 'Australia's disappearing market for life annuities', *UNSW Centre for Pensions and Superannuation Discussion Paper*, **Discussion Paper 01/08**.

Habib, F., Huang, H., Mauskopf, A., Nikolic, B. and Salisbury, T. (2020), 'Optimal Allocation to Deferred Income Annuities', *Insurance: Mathematics and Economics*, **90**, 94-104.

Henry, K. (2009), 'Australia's Future Tax System: The Retirement Income System: Report on Strategic Issues', Commonwealth of Australia.

Horneff, V., Maurer, R. and Mitchell, O.S. (2020), 'Putting the pension back in 401 (k) retirement plans: Optimal versus default deferred longevity income annuities', *Journal of Banking & Finance*, 105783.

Horneff, W., Maurer, R. and Rogalla, R. (2010), 'Dynamic portfolio choice with deferred annuities', *Journal of Banking & Finance*, **34**, 2652-2664.

Huang, H., Milevsky, M.A. and Salisbury, T.S. (2012), 'Optimal retirement consumption with a stochastic force of mortality', *Insurance: Mathematics and Economics*, **51**, 282-291.

Huang, H., Milevsky, M.A. and Young, V.R. (2017), 'Optimal purchasing of deferred income annuities when payout yields are mean-reverting', *Review of Finance*, **21**, 327-361.

Hulley, H., Mckibbin, R., Pedersen, A. and Thorp, S. (2013), 'Means-Tested Public Pensions, Portfolio Choice and Decumulation in Retirement', *Economic Record*, **89**, 31-51.

Iskhakov, F., Thorp, S. and Bateman, H. (2015), 'Optimal annuity purchases for Australian retirees', *Economic Record*, **91**, 139-154.

Jin, H. and Yu Zhou, X. (2008), 'Behavioral portfolio selection in continuous time', *Mathematical Finance: An International Journal of Mathematics, Statistics and Financial Economics*, **18**, 385-426.

Konicz, A.K. and Mulvey, J.M. (2013), 'Applying a stochastic financial planning system to an individual: Immediate or deferred life annuities?', *The Journal of Retirement*, **1**, 46-60.

Kotlikoff, L.J. (2008), 'Economics' Approach to Financial Planning', Unpublished paper.

Lockwood, L.M. (2012), 'Bequest motives and the annuity puzzle', *Review of Economic Dynamics*, **15**, 226-243.

Lockwood, L.M. (2018), 'Incidental bequests and the choice to self-insure late-life risks', *American Economic Review*, **108**, 2513-2550.

Malloy, L.C. (2015), 'Loss aversion, education, and intergenerational mobility', *Education Economics*, **23**, 318-337.

Merton, R.C. (1969), 'Lifetime portfolio selection under uncertainty: The continuous-time case', *The Review of Economics and Statistics*, **51**, 247-257.

Modigliani, F. and Brumberg, R. (1954), 'Utility analysis and the consumption function: An interpretation of cross-section data', G. Allen, London.

O'Meara, T., Sharma, A. and Bruhn, A. (2015), 'Australia's Piece of the Puzzle–Why Don't Australians Buy Annuities?', *Australian Journal of Actuarial Practice*, **3**, 47-57.

Pashchenko, S. (2013), 'Accounting for non-annuitization', *Journal of Public Economics*, **98**, 53-67.

Polkovnichenko, V. (2007), 'Life-cycle portfolio choice with additive habit formation preferences and uninsurable labor income risk', *The Review of Financial Studies*, **20**, 83-124.

Price, K., Storn, R.M. and Lampinen, J.A. (2006), 'Differential evolution: a practical approach to global optimization', Springer Science & Business Media.

R Core Team (2019), 'R: A language and environment for statistical computing', Vienna: R Foundation for Statistical Computing.

Rabin, M. and Thaler, R.H. (2001), 'Anomalies: risk aversion', *Journal of Economic Perspectives*, **15**, 219-232.

Samuelson, P.A. (1969), 'Lifetime Portfolio Selection by Dynamic Stochastic Programming', *The Review of Economics and Statistics*, **51**, 239-246.

Thorp, S., Kingston, G. and Bateman, H. (2007), 'Financial engineering for Australian annuitants', Edward Elgar, Cheltenham.

Tversky, A. and Kahneman, D. (1992), 'Advances in prospect theory: Cumulative representation of uncertainty', *Journal of Risk and Uncertainty*, **5**, 297-323.

van Bilsen, S. and Laeven, R.J.A. (2020), 'Dynamic consumption and portfolio choice under prospect theory', *Insurance: Mathematics and Economics*, **91**, 224-237.

Yaari, M.E. (1965), 'Uncertain lifetime, life insurance, and the theory of the consumer', *The Review of Economic Studies*, **32**, 137-150.

Yogo, M. (2016), 'Portfolio choice in retirement: Health risk and the demand for annuities, housing, and risky assets', *Journal of Monetary Economics*, **80**, 17-34.

Appendix

(a) Choice of cameos

The key dimensions we aim to span in the seven cameos in Table 1 include balance and homeownership as reflects wealth or "financial means", and financial preferences. We investigate three initial balance levels of \$200,000 (low), \$500,000 (average) and \$800,000 (high) with reference to account balance statistics[21], allowing for the fact that balances are likely to grow as the superannuation system matures further. We investigate both renters and homeowners at low and average wealth levels, but assume that those at the high wealth level are always homeowners. Preferences are characterised by either loss aversion utility or constant relative risk aversion (CRRA) utility, capturing a desire to achieve a specified consumption target and maximise the overall level of consumption respectively. Two consumption targets are tested corresponding to the "Modest" and "Comfortable" budget standards at September 2019 as formulated by the Association of Superannuation Funds of Australia (2019) (ASFA), with Modest applied at low balance and Comfortable at average and high balances. CRRA utility is considered only for average and high wealth homeowners. Overall, we test 14 scenarios comprised of seven cameos each with 'weak' and 'strong' preference parameterisations.

Homeownership status specifically impacts the consumption target under loss aversion utility. Homeowners are assumed to have consumption targets consistent with the ASFA standards, which are formulated for

[21] These round numbers are based on account-balance statistics from the Association of Superannuation Funds of Australia (Clare, 2019).

homeowners. Renters are treated by adjusting their consumption target upwards for assumed rental costs less the housing costs embedded in the ASFA standards, and allowing for access to rental assistance. Our treatment is to assume that home prices and rents paid increase with account balance (i.e. those with higher wealth occupy a more expensive home), with home price data drawn from the CoreLogic Hedonic Home Value Index for September 2019. We use median values for regional units at low wealth, median house across all capitals at average wealth, and median Sydney house at high wealth. Rents are determined by applying annual rental yields of 5.35% for the low wealth and 3.83% for average wealth, with these yields based on the September 2019 CoreLogic Quarterly Rent Review.

Under loss aversion utility as per Equation (6), the parameters of Tversky and Kahneman (1992) are used for weak loss aversion and those of Blake et al. (2013) are used for strong loss aversion. For CRRA utility as per Equation (8) we test parameterisations of ρ = 2 (as used by Iskhakov et al. (2015)) for weak risk aversion, and ρ = 5 (as used in some studies such as Azar (2006)) for strong risk aversion. For bequest motives, we assume ϕ = 0.5, as per Malloy (2015) . This is a relatively low bequest motive, reflecting an assumption that consumption is of greater concern than bequests for the individuals being examined. We vary this under sensitivity testing.

(b) Minimum Drawdown and Age Pension rules

Minimum drawdown rules apply for D_t in Equations (1) and (2), with the minimum proportion of A_t calculated as per Table A1.

Table A1 – Minimum Drawdowns for Account-Based Pensions in Australia

Age	Minimum Drawdown of Balance
Up to 65	4%
65-74	5%
75-79	6%
80-84	7%
85-89	9%
90-94	11%
95 and above	14%

Figures sourced from Australian Taxation Office (2020)

The Age Pension rules are based on the structure and thresholds in place at September 2019, assuming no assets outside of homeownership, the account-based pension (ABP), life annuity (LA) and deferred life annuity (DLA). The total Age Pension payment P_t in Equation (4) is the sum of four components: the base amount BP_t, pension supplement SP_t, energy supplement EP_t, and rental assistance RP_t (if applicable)[22]. BP_t is dependent on two means tests, an asset test BPA_t and an income test BPI_t. The calculations are as follows:

[22] Rental assistance is not a part of the Age Pension, but for notational simplicity we include it in P_t.

$$BPA_t = BP_{t,max} - 0.078(PA_t - MPA) \tag{10}$$

$$BPI_t = BP_{t,max} - 0.5(PI_t - \$4{,}524) \tag{11}$$

$$BP_t = \max\big(0, \min(BPA_t, BPI_t)\big) \tag{12}$$

where $BP_{t,max}$ is the maximum base pension payable and is equal to \$22,110.40 at t = 0 and is increased by 1.5% in each subsequent year[23]. *MPA* is equal to \$263,250 for homeowners and \$473,750 for renters. PA_t and PI_t represent the assessable assets and income for the purposes of asset test and income test respectively and are calculated as follows:

$$PA_t = A_t - I_{t=0}[L + DL] + 0.6I_{t<19}[L + DL] + 0.3I_{t\geq 19}[L + DL] \tag{13}$$

$$PI_t = 0.01A_t + 0.03\max[0, A_t - \$51{,}800] + 0.6\left[\frac{L}{a_L} + I_{t\geq 18}\frac{DL}{a_{DL}}\right] \tag{14}$$

SP_t, EP_t and RP_t are conditional on receiving some base pension (i.e. $BP_t > 0$) and are calculated as:

$$SP_t = \$962 + \frac{BP_t}{22{,}110.4}(\$1791.4 - \$962) \tag{15}$$

$$EP_t = \$366.6 \tag{16}$$

$$RP_t = \begin{cases} 0 & if\ R_t < \$3{,}203.2 \\ 0.75\ (R_t - \$3{,}203.2) & if\ \$3{,}203.2 \leq R_t \leq \$7{,}987.2 \\ \$3{,}588 & if\ R_t > \$7{,}987.2 \end{cases} \tag{17}$$

where R_t is defined in Table 1.

(c) Economic assumptions

Following on from Section II(d), the real growth rate of 1.5% per annum is in line with estimates of long-term labour productivity growth rate (see Table 1 of Australian Government Productivity Commission (2020)), and compares with historical growth of 1.4% in Male Average Weekly Ordinary Time Earnings (MAWOTE) above the CPI between 1994 and 2019.

Return series for the growth portfolio and defensive portfolio are constructed by combining historical return series for selected asset classes[24] spanning the period December 1984 to June 2019. The growth portfolio comprises 44% in Australian equities, 44% in world equities and 12% in listed property. The defensive portfolio comprises 40% in Australian fixed income, 40% in world fixed income and 20% in Australian cash. We then

[23] This is equivalent to the assumed increase in Male Average Weekly Ordinary Time Earnings (MAWOTE) in excess of the Consumer Price Index (CPI) in our modelling. All other dollar values in Equations (10) – (17) are indexed at CPI. See Section II(d) and Appendix (c) for further details on how the model allows for indexation.

[24] Asset return series are based on the S&P/ASX300 Accumulation Index for Australian equities, the MSCI World Ex. Australia Index in A\$ for world equities, the S&P/ASX 300 Property Trust (REIT) Accumulation Index for listed property, the Citi Australian Bond Accumulation Index for Australian fixed income, the Citi World Bond Accumulation (A\$ Hedged) Index for world fixed income, and returns on bank-accepted bills for Australian cash. Returns are deflated by the Australian Consumer Price Index.

mean-adjust the return series so that the growth portfolio generates a compound real return of 4.5% per annum, while the defensive portfolio generates a compound real return of 1.0% per annum. These mean returns are set with reference to historical returns on world equities and fixed income (Dimson et al., 2019). This approach retains the higher moments and correlation structure of the historical return series, with standard deviations of 15.0% per annum for the growth portfolio and 4.4% per annum for the defensive portfolio. The output from this process is a sample of 135 annual returns (quarterly rolling) that are used in forming expectations under Equations (1) and (2).